

Text S1

1. Numerical simulation setup

The computational method used here to study the evolution of phytoplankton at fronts solves the non-hydrostatic rotating Navier-Stokes equations in two-dimensional slices. Although variations are neglected in the direction normal to the computational domain, the velocity field retains all three components, and evolves according to

$$\frac{\partial u}{\partial t} + \mathbf{u}_T \cdot \nabla u - f v = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu \nabla^2 u, \quad (\text{S1})$$

$$\frac{\partial v}{\partial t} + \mathbf{u}_T \cdot \nabla v + w \frac{dV_G}{dz} + f u = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \nu \nabla^2 v, \quad (\text{S2})$$

$$\frac{\partial w}{\partial t} + \mathbf{u}_T \cdot \nabla w = -\frac{1}{\rho_0} \frac{\partial p}{\partial z} + b + \nu \nabla^2 w, \quad (\text{S3})$$

$$\frac{\partial b}{\partial t} + \mathbf{u}_T \cdot \nabla b + u M^2 = \kappa \nabla^2 b, \quad (\text{S4})$$

$$\nabla \cdot \mathbf{u} = 0, \quad (\text{S5})$$

where f is the Coriolis frequency. The two-dimensional computational domains are arranged as in Figure 2 so that the gradient operator is $\nabla \equiv (\partial/\partial x, 0, \partial/\partial z)$ in Domains 1 and 2, and $\nabla \equiv (0, \partial/\partial y, \partial/\partial z)$ in Domain 3. Here, x and y are the cross-front and along-front directions, respectively. For the simulations in Domains 2 and 3, the front is

included by imposing a constant background buoyancy gradient, M^2 , while $M^2 = 0$ in Domain 1. The buoyancy, b , is then defined as the departure from the background state

$$b = b_T - M^2 x, \quad (\text{S6})$$

where b_T is the total buoyancy. Similarly, the total velocity \mathbf{u}_T is decomposed into departures \mathbf{u} from a flow V_G in thermal wind balance with M^2 :

$$\mathbf{u} = \mathbf{u}_T - V_G \hat{\mathbf{j}}, \quad V_G \equiv M^2 / f. \quad (\text{S7})$$

The viscosity and diffusivity, ν and κ , are both set to $2 \times 10^{-4} \text{m}^2/\text{s}$, which maintains numerical stability and is much smaller than the turbulent diffusivity in the SBL in each case. The buoyancy is initialized with a mixed layer depth of $H = 150\text{m}$ with a constant stratification below:

$$b_T(t=0) = \begin{cases} -HN_0^2 + M^2 x & \text{if } -H < z < 0, \\ N_0^2 z + M^2 x & \text{if } z \leq -H, \end{cases} \quad (\text{S8})$$

where $N_0 = 9 \times 10^{-5} \text{s}^{-2}$. The frontal strength is set to $M^2 = 4.24 \times 10^{-7} \text{s}^{-2}$ in Domains 2 and 3, and $M^2 = 0$ in Domain 1. Assuming that the buoyancy gradient is controlled by the temperature (with constant salinity), and using a thermal expansion coefficient of $\alpha = 1.7 \times 10^{-4} \text{C}^{-1}$, this frontal strength corresponds to a temperature gradient of about $0.25^\circ \text{C}/\text{km}$.

The physical model is coupled with an idealized phytoplankton model,

$$\frac{\partial P}{\partial t} + \mathbf{u} \cdot \nabla P = \mu_0 e^{z/h_l} P - mP + \kappa \nabla^2 P, \quad (\text{S9})$$

where $\mu_0 = 1 \text{day}^{-1}$ is the maximum growth rate at $z = 0$, $m = 0.1 \text{day}^{-1}$ is a constant loss rate, $h_l = 10\text{m}$, and $\kappa = 2 \times 10^{-4} \text{m}^2/\text{s}$ is a weak diffusion added to maintain numerical stability. Based on these parameters, Sverdrup's critical depth is $H_c = (\mu_0/m)h_l = 100\text{m}$,

and less than the mixed layer depth, $H = 150\text{m}$. The initial phytoplankton profile is

$$P(t=0) = \begin{cases} 1 & \text{if } -H < z < 0, \\ 0 & \text{if } z \leq -H. \end{cases} \quad (\text{S10})$$

The turbulent diffusivity κ_T was calculated from the resolved vertical turbulent flux and the mean phytoplankton gradient in each simulation:

$$\kappa_T = \frac{-\langle w'P' \rangle}{\partial \langle P \rangle / \partial z}. \quad (\text{S11})$$

Here, the angled brackets denote an average over the full horizontal extent, the upper 100m in the vertical, and for one inertial period in time ending at $t = 6\text{days}$. Primes denote a departure from this average, e.g. $P' = P - \langle P \rangle$. An approximate expression for the critical turbulent diffusivity is given in equation (16) of *Taylor and Ferrari* [2010],

$$\kappa_c \simeq \frac{h_l^2}{m}(\mu_0 - m)^2. \quad (\text{S12})$$

Phytoplankton growth is expected when $\kappa_T \leq \kappa_c$, and in our simulations $\kappa_c \simeq 9 \times 10^{-3}\text{m}^2/\text{s}$.

2. Scaling of restratification by frontal instability

The stratification in the upper ocean is determined by a competition between restratification and mixing. As discussed in *Thomas and Ferrari* [2008], restratification can be driven by several processes including frictional effects, frontogenesis, and the potential energy release associated with frontal instabilities. Here, we focus on restratification associated with mixed layer baroclinic and symmetric instabilities as discussed in the main text. When restratification driven by the frontal instability is able to overcome a destabilizing atmospheric forcing, the mixed layer will become stably stratified, suppressing vertical mixing, and possibly triggering a phytoplankton bloom. In the simulations pre-

sented in this paper, the frontal strength and surface forcing have been selected so that symmetric and baroclinic instabilities are both able to restratify the mixed layer. In order to see how the restratification by frontal instabilities compare to the surface forcing in other conditions, it is helpful to review scalings for mixed layer restratification developed in previous studies. It should be noted that a general scaling for the vertical turbulent diffusivity as a function of the frontal strength and surface forcing remains unknown. Although it is beyond the scope of this paper, parameterizing the vertical mixing at fronts will be an important step towards improving ocean models and their representation of primary production. Since the process of restratification is different for symmetric and baroclinic instability, we consider each separately.

2.1. Baroclinic Instability

The baroclinic instability of mixed layer fronts (commonly referred to as ‘mixed layer instability’, or MLI) was studied in detail by *Boccaletti et al.* [2007]. The instability extracts potential energy from the front, leading to a slumping of the isopycnal surfaces and restratification. *Fox-Kemper and Ferrari* [2008] and *Fox-Kemper et al.* [2008] derived a scaling for the rate of restratification by MLI. The restratification by MLI can be compared to the destabilizing effect of surface forcing by comparing the associated vertical buoyancy fluxes. According to the scaling, the maximum vertical buoyancy flux associated with MLI is:

$$\overline{w'b'}_{MLI} = C_{MLI} \frac{H^2(\nabla b)^2}{|f|}, \quad (\text{S13})$$

where $C_{MLI} \simeq 0.06$ is an empirically determined scaling constant. The stabilizing effect of MLI competes against the destruction of buoyancy through a surface buoyancy flux,

B_0 , and an ‘Ekman buoyancy flux’ (EBF) associated with an unstable cross-front Ekman flow, $EBF = \rho_0^{-1} \boldsymbol{\tau} \cdot (f^{-1} \hat{\mathbf{k}} \times \nabla b)$. The latter is important when the wind stress, $\boldsymbol{\tau}$, has a component in the direction of the thermal wind [Thomas and Lee, 2005]. A stability parameter can be defined as the ratio of the stabilizing and destabilizing buoyancy fluxes:

$$R_{MLI} = C_{MLI} \frac{H^2 (\nabla b)^2}{|f| (B_0 + EBF)}. \quad (S14)$$

When $R_{MLI} \ll 1$, the surface forcing is strong relative to the frontal restratification, and the mixed layer is expected to remain unstratified, while stratification can develop when $R_{MLI} \gg 1$. For the parameters used in this study, $R_{MLI} \simeq 57$, consistent with the fact that the mixed layer restratifies in Domain 3.

2.2. Symmetric Instability

The restratification by symmetric instability (SI) is significantly different than MLI. The stability criteria for SI can be expressed in terms of the potential vorticity: $PV = (\mathbf{f} + \boldsymbol{\omega}) \cdot \nabla b$ where $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ is the relative vorticity. SI develops when $PV < 0$ [Hoskins, 1974] and quickly brings the PV to zero by restratifying the mixed layer (by increasing $f \partial_z b$) as seen in Domain 2 and in Taylor and Ferrari [2010]. SI grows faster than baroclinic instability in most cases when $PV < 0$ [Stone, 1970] and dominates the initial phases of frontal instability. However when $PV \geq 0$ the symmetric instability stops. There is recent observational evidence that SI can also maintain a stable stratification under very intense atmospheric forcing [D’Asaro et al., 2011]. Unlike MLI, which restratifies the entire mixed layer, SI is only capable of generating significant stratification below a ‘convective layer’, $z < -h$. Taylor and Ferrari [2011] presented a scaling for the convective layer depth, h , in terms of the frontal strength, $M^2 = |\nabla b|$, the depth of the unstable SI layer, H , and

the surface wind and buoyancy forcing. The ratio of the convective layer depth to the boundary layer depth, h/H , determines the relative importance of SI. When $h/H \ll 1$, a large fraction of the boundary layer will become stratified as a result of SI, while in the limit when $h/H \rightarrow 1$, the boundary layer will be unstratified. For a given set of parameters, the ratio h/H can be readily found by numerically solving the following fourth order polynomial equation (see *Taylor and Ferrari* [2011]):

$$\left(\frac{M^4}{f^2}\right)^3 \left(\frac{h}{H}\right)^4 - C_{SI} \frac{B_0 + EBF}{H^4} \left(1 - \frac{h}{H}\right)^3 = 0. \quad (\text{S15})$$

where $C_{SI} \simeq 14$ is an empirical scaling constant, and the small entrainment coefficients have been neglected. Using the parameters from the simulations shown in Figure 3, $h/H \simeq 0.035$, indicating that SI is expected to develop and restratify most of the mixed layer, as is indeed seen in Domain 2.

References

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