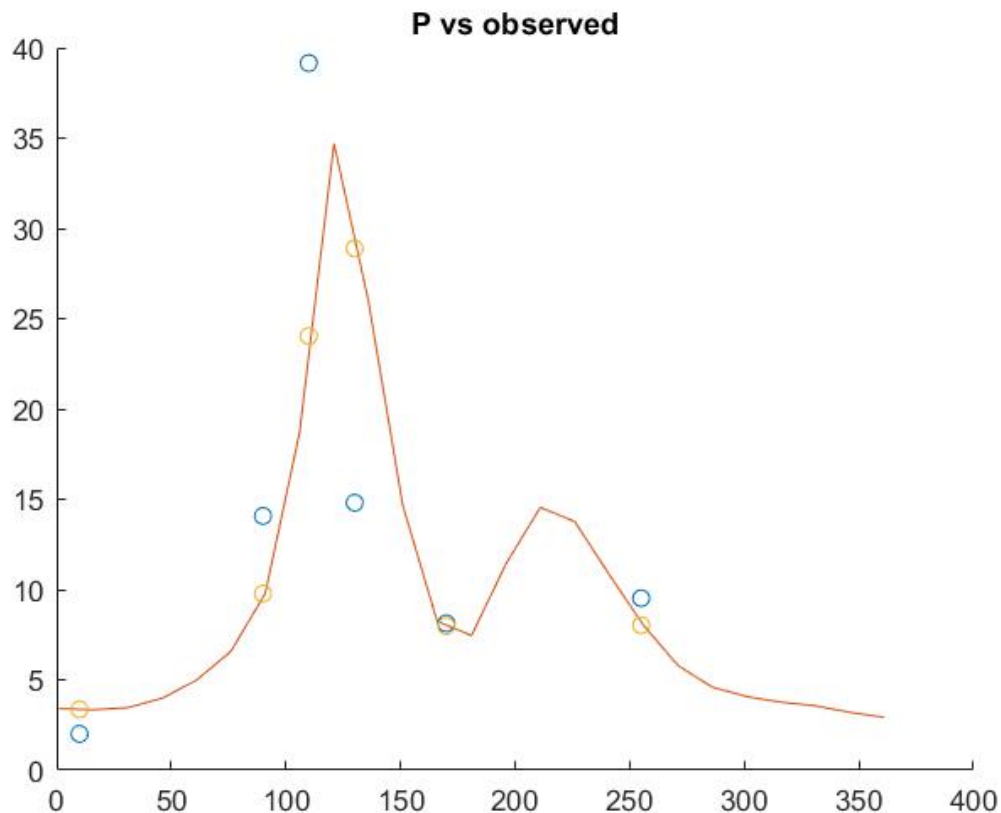


## Problem Set 1

1. I was able to reproduce Riley's results using the following code, which produced the plot of phytoplankton concentrations below, across time in days. By extrapolating values of  $P$  along the model line and comparing them to the observed values, I obtained a percent error value of 41.6%.

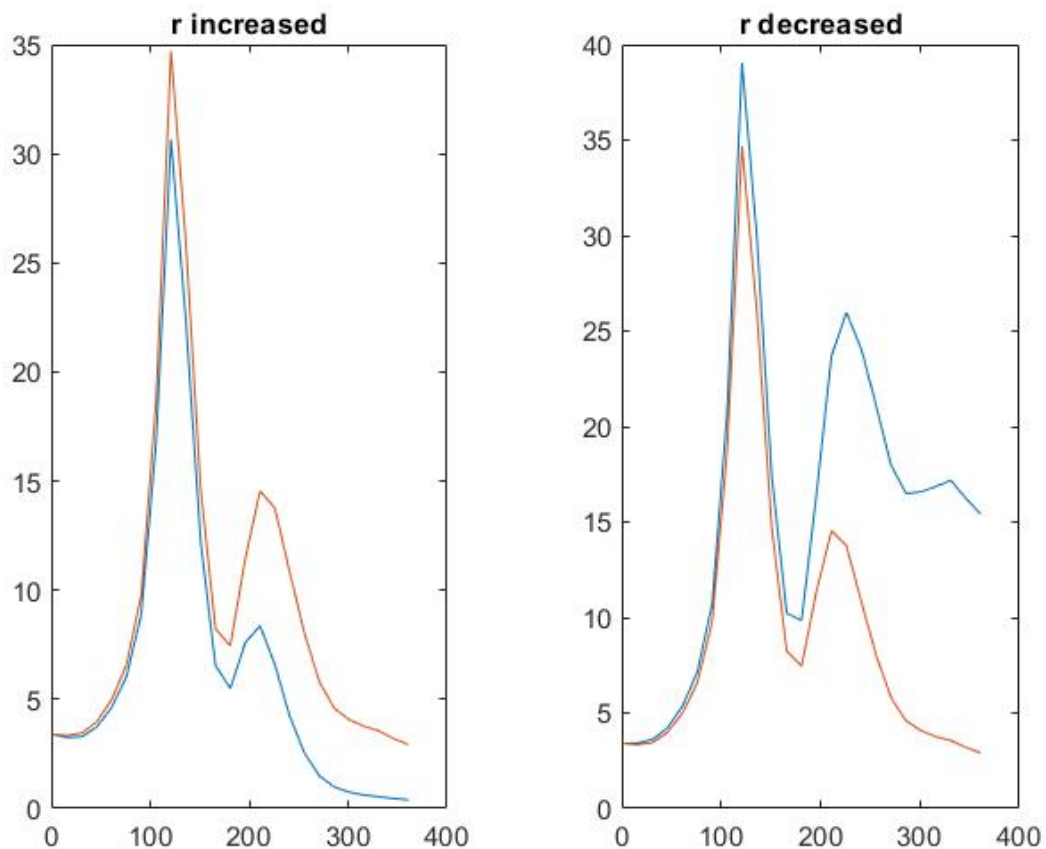
```
%constants
Ro = 0.0175;
r = 0.069;
g = 0.0075;
p = 2.5;
P = zeros(24,1)
P(1) = 3.4; %Po
for t=1:24
    RT(t) = Ro*exp(r*T(t));
    G(t) = g*Z(t);
    Ph(t) = ((p*I(t))/(k(t)*z1(t)))*(1-exp(-k(t)*z1(t)))*OneMinusN(t)*OneMinusV(t);
    P(t+1) = P(t)*exp(15*(Ph(t) - RT(t) - G(t)));
end
```



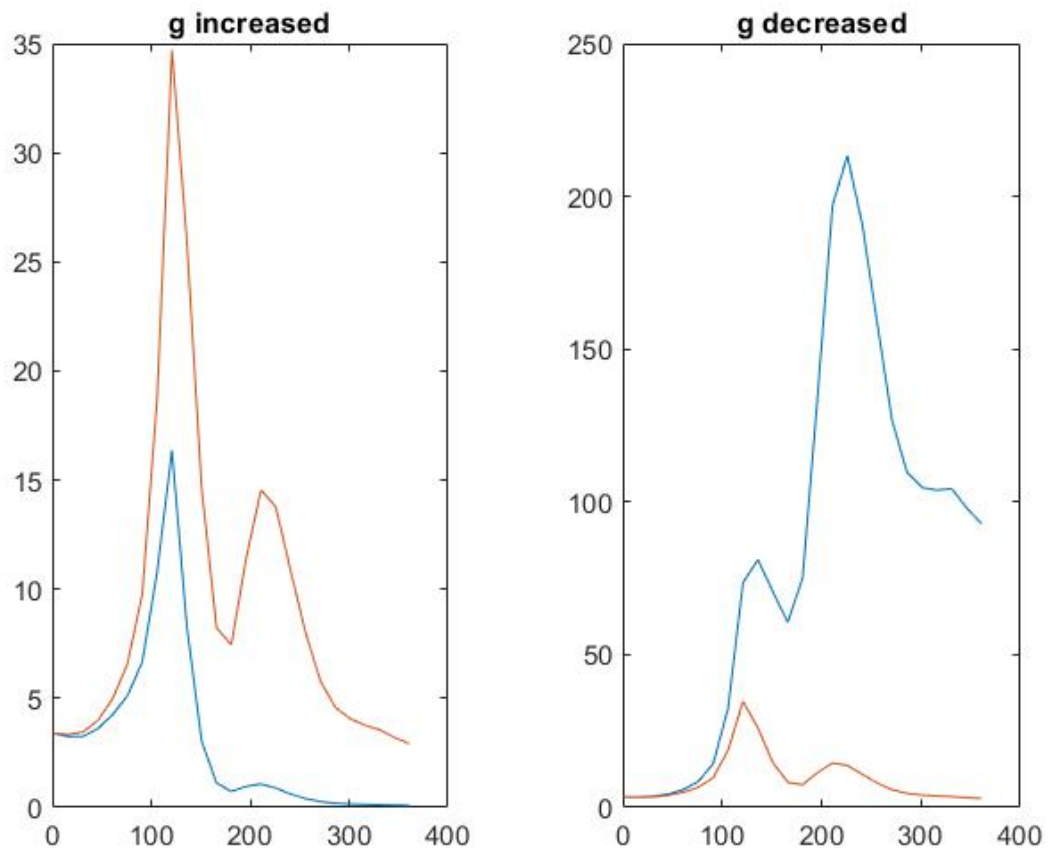
### Sensitivity analysis:

The red line in each plot is the original distribution with the parameters set as given. The blue lines represent an increase of 20% in the given parameter (left) and a decrease of 20% (right). This analysis allowed us to see how strongly and in what ways each parameter impacted the system. I calculated the average percent error between the original model values and the increased and decrease values, as well as the percent error between the increased and decreased values with the observational data. By having both we can understand both the sensitivity of the model, and the effectiveness of the model in fitting to the data with slight fluctuations. The errors listed in parentheses refer to the error against observed values.

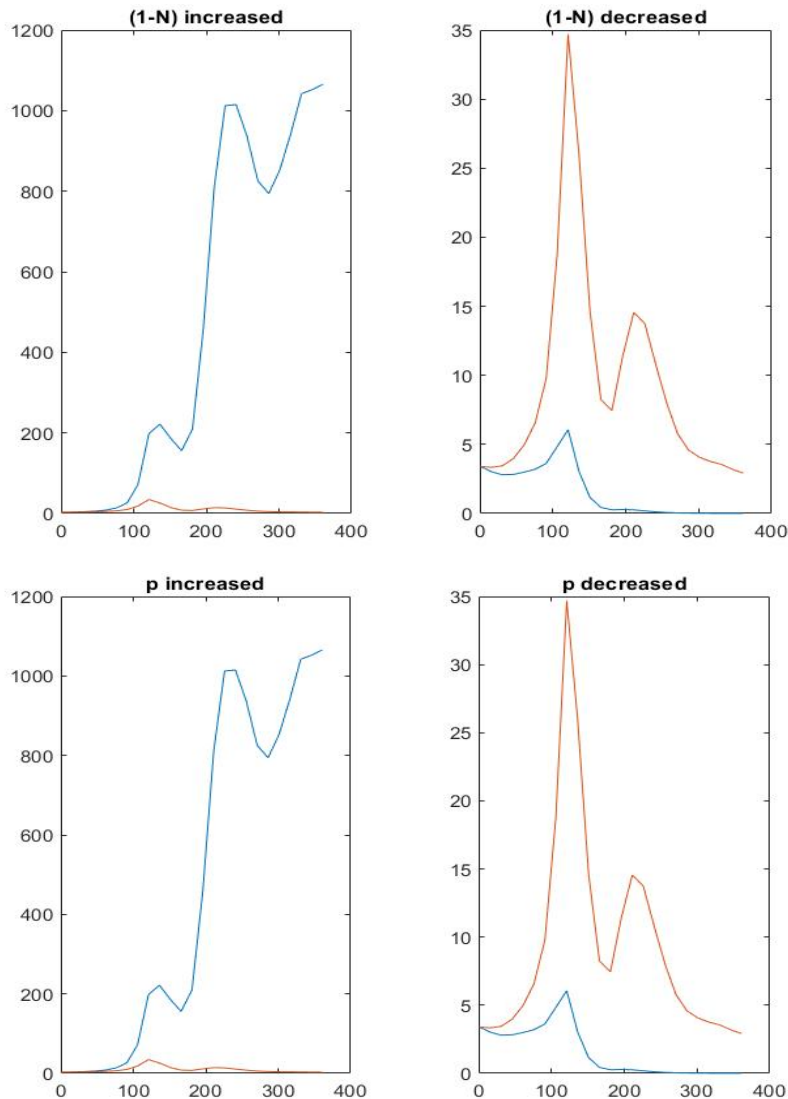
The variable  $r$  is a constant representing the rate of change of the respiratory rate with temperature. The given value, 0.069, represents a doubling with an increase in  $10^{\circ}\text{C}$ . This value is used in calculating  $R_T$ , which is the respiratory rate at a given temperature,  $T$ . When I increased  $r$  by 20% (left, blue line), the initial growth of the phytoplankton population remained the same, and throughout the rest of the year, fell below the original value, but only slightly. It did, however, cause an almost complete die out by the end of the year. When I decreased  $r$  by 20% (right, blue), there was again minimal difference in the beginning of the year, followed by an opposite reaction of a consistently larger than normal  $P$  throughout the rest of the year. Of all the parameters tested, this actually had the least impact on the model overall, with average percent error being only 39% (52%) and 119% (66%) respectively.



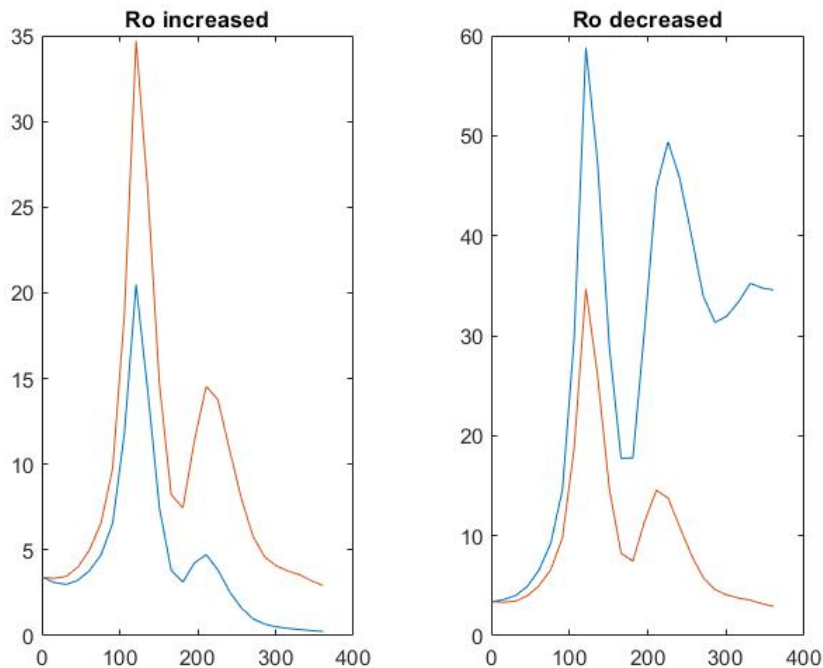
The variable  $g$  is grazing by zooplankton, which when multiplied by the quantity of zooplankton,  $Z$ , gives the rate of grazing,  $G$ . Increasing  $g$  (left, blue line) resulted in a decreased phytoplankton concentration, which compounded with time. As more and more grazing took place, the phytoplankton population couldn't keep up and effectively crashed by mid summer. When decreased, however, the phytoplankton took off, released from much of the top down control of grazing. There was a huge difference in the relative difference in percent error between increased and decreased grazing. The error for increased grazing was only 66% (66%), while the decrease caused an error of 1125% (465%) as growth increased unchecked by grazing.



The **(1-N)** value in the phytoplankton concentration equation is used in calculating the photosynthetic rate where N is the reduction in rate due to nutrient depletion. Similarly, **p**, the photosynthetic constant, has the same influence on photosynthetic rate. Therefore, changing both parameters influence P the same way. With increased (1-N) or p (left, blue), there is a huge increase in phytoplankton concentration. By increasing (1-N) we are **decreasing the rate of nutrient depletion** – and therefore increasing the potential for photosynthesis. Likewise, increasing photosynthetic rate, increases overall photosynthesis. In both scenarios, phytoplankton increases massively, causing a percent error 8818% (2246%). When both are decreased (right, blue), we end up with a percent error of only 77% (81%), but a full collapse of the system again by mid summer, so biologically just as significant.



Finally, the variable  $R_0$  represents the respiratory rate at  $0^\circ\text{C}$ , which impacts the respiratory rate  $R(t)$ . When I increased  $R_0$  (left, blue), there was a subsequent decrease in  $P(t)$ , but the pattern held and we didn't have die off until the very end of the year. The percent error was only 55% (54%). When we decreased it however (right, blue), there was an increase in phytoplankton (since their overall respiration rates were lower) and an error of 303% (127%). In this case, the pattern again held overall, and we didn't have a "runaway" phytoplankton concentration as we did with some of the other parameters.



Overall, the model was most sensitive to changes in a parameters that caused increases in phytoplankton concentration (increases in  $p$ ,  $1-N$ , and decreases in  $R_0$ ,  $g$ , and  $r$ ). The parameter with the least influence overall was  $r$  (especially when increased), while the most influential seemed to be both  $p$  and  $1-N$  (especially when increased). Riley suggests that some of the most important factors are solar radiation, temperature, transparency of water, and depth of isothermal surface layer, zooplankton, and phosphate. Based on these data, I feel that respiration rate was not as influential as photosynthetic rate, as far as the model is concerned. The depth of the isothermal surface layer and phosphate concentrations make a lot of sense since they so strongly impact nutrient availability which the model was extremely sensitive to. My one addition would be to stress the importance of the given photosynthetic constant.

2. For  $P$  to be periodic, the value of  $P(25)$  must equal the value of  $P(0)$ . The value of  $g$  that ensures this is 0.0074348. When we vary other parameters, the end condition will rarely meet the beginning condition, so the  $g$  value would need to change with each of these parameters. When we shift the values as we did before, the  $g$  value would have to be entirely different:

Parameter	20% increase	20% decrease
$R_0$	.006364	.008506
(1-N) and p	.009993	.004877
r	.006572	.008156

As expected, the change needed is less for  $R_0$  and  $R$  than it is for (1-N) and p.

- When we set  $g$  to 0.004348, but change the other parameters, we do not have perfect periodicity, since the 20% variation in  $Z$  leads to changes in the end condition of  $P$ . But, it does allow for relatively stable periodicity. We see that the general pattern of growth year after year remains the same, with fluctuations on a yearly basis in total growth throughout the year. Each time the simulation is run, the output looks a little bit different, but most had variation similar to the plot below. This is an effective way of gaining an understanding of the dynamics between parameters.

