Figure 1. Duplication of results from Riley of the theoretical model prediction of P vs the observations. (Further analysis in the comments of the code)
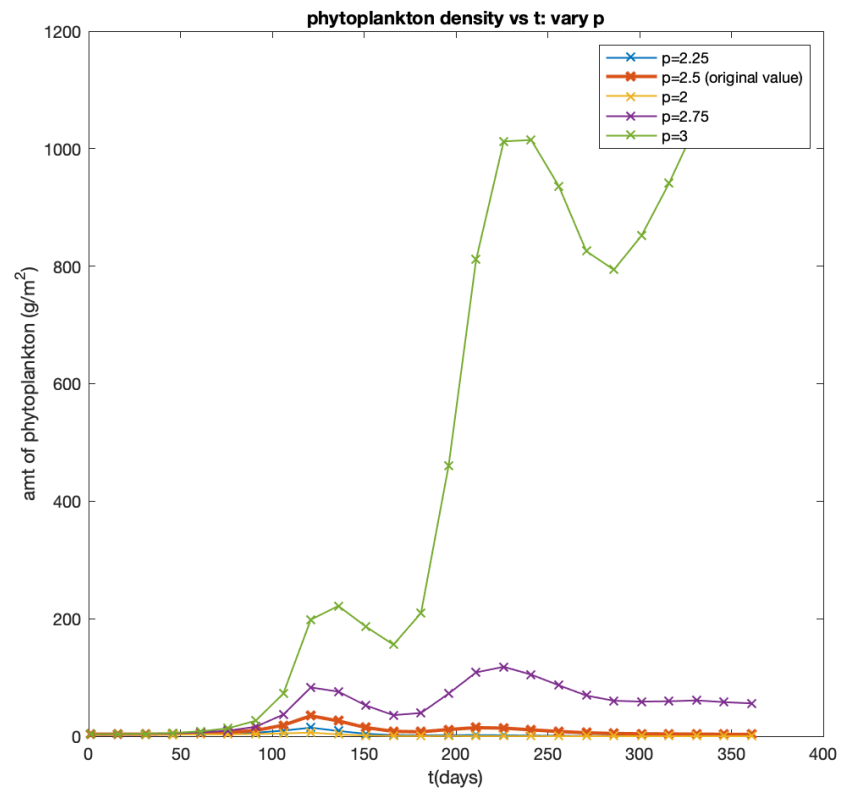
Figure 2. Sensitivity analysis: P over the course of a year, changing the photosynthesis amplifying parameter p 10% and 20%. The Red curve indicates the original parameter choices. (further analysis in comments of code)
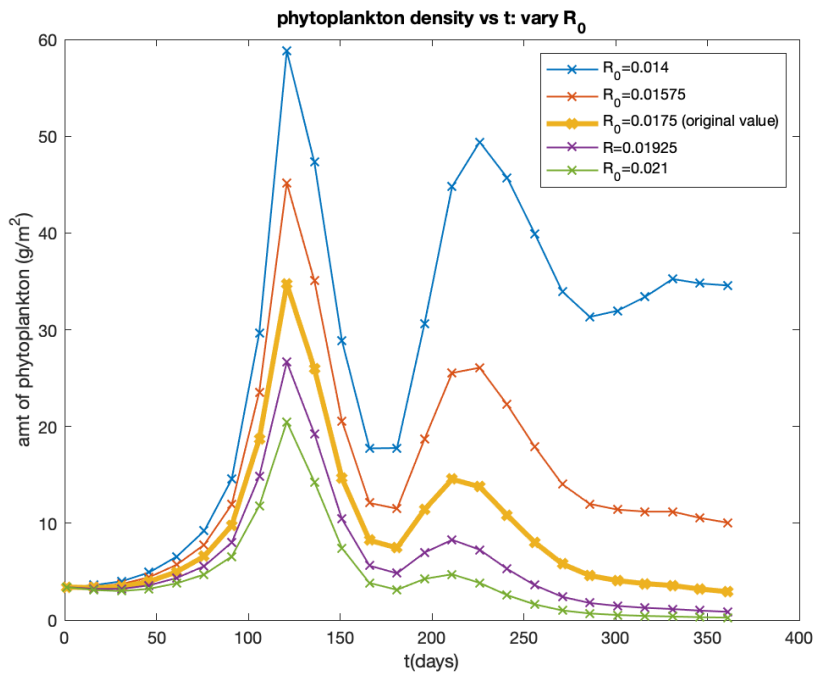
Figure 3. Sensitivity analysis: P over the course of a year, changing the temperature-dependent amplifying parameter R_0 10% and 20%. The yellow curve indicates the original parameter choices. Note that the deviations from the control parameter P in R_0 are relatively small compared to those in p, suggesting that P is very sensitive to choices of p but not as sensitive to R_0. (further analysis in comments of code)
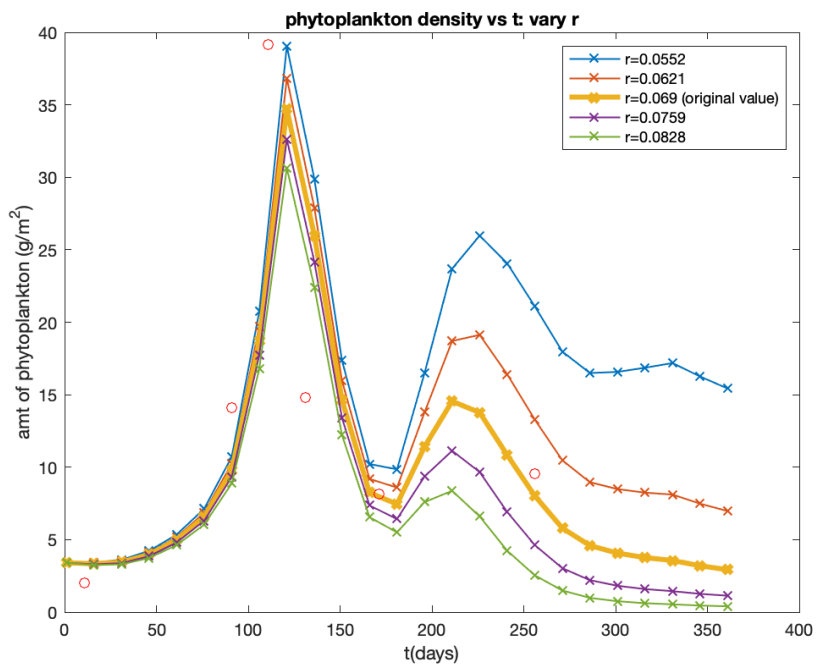
Figure 4. Sensitivity analysis: P over the course of a year, changing the temperature-dependent exponential parameter r_0 10% and 20%. The yellow curve indicates the original parameter choices. Note that the deviations from the control parameter P in R_0 are relatively small compared to those in p (even closer to the control case than deviations in R_0), suggesting that P is very sensitive to choices of p but not as sensitive to r. (further analysis in comments of code)
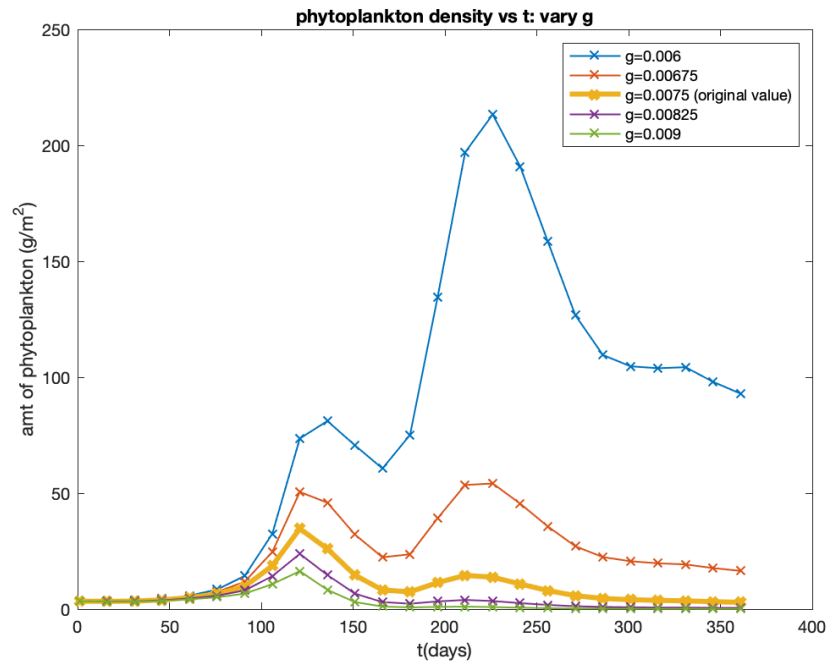
Figure 5. Sensitivity analysis: P over the course of a year, changing the grazing parameter g 10% and 20%. The yellow curve indicates the original parameter choices. Note that the deviations from the control parameter P in g are rather large compared to those in R_0 (though not nearly as sensitive as p, the reason being that there is also a temperature-dependent term which aids in the decay of P), suggesting that P is still quite sensitive to choices of g but not as sensitive to r. (further analysis in comments of code)

phytoplankton density vs t: vary nutrient depletion term 1-N for one time interval
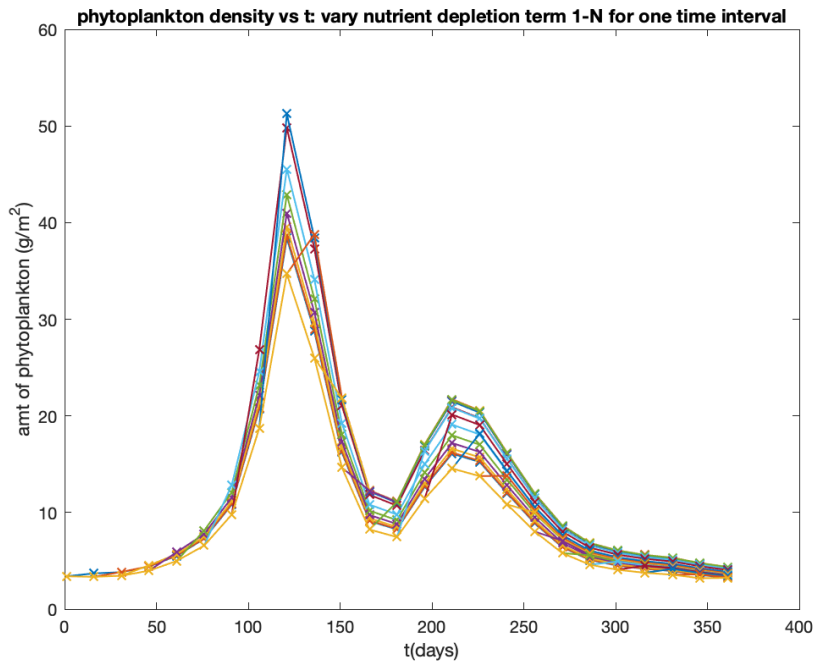
Figure 6 (supplementary figure). Sensitivity analysis: P over the course of a year, increasing the nutrient depletion parameter 1-N by 20% for a given time interval (it is a redundant calculation to uniformly amplify or diminish 1-N by 10% or 20% in the sensitivity analysis). In other words, each curve corresponds to a time interval for which we increase the corresponding 1-N value there by 20%. Although a bit muddled in the steeper gradient sections of this plot, looking near the local peaks of P we can observe that the sensitive P values deviate from their standard values by about 5 (first peak) or 10 (secondary peak) (further analysis in comments of code). This is suggestive of the sensitivity of P on 1-N, though the obvious conclusion is reached upon uniformly enhancing 1-N, which has the same degree of sensitivity of p.

Commented [11]: The problem was intended to be the uniform increase / decrease to 1-N which you correctly identified as identical to the earlier sensitivity of p.

1
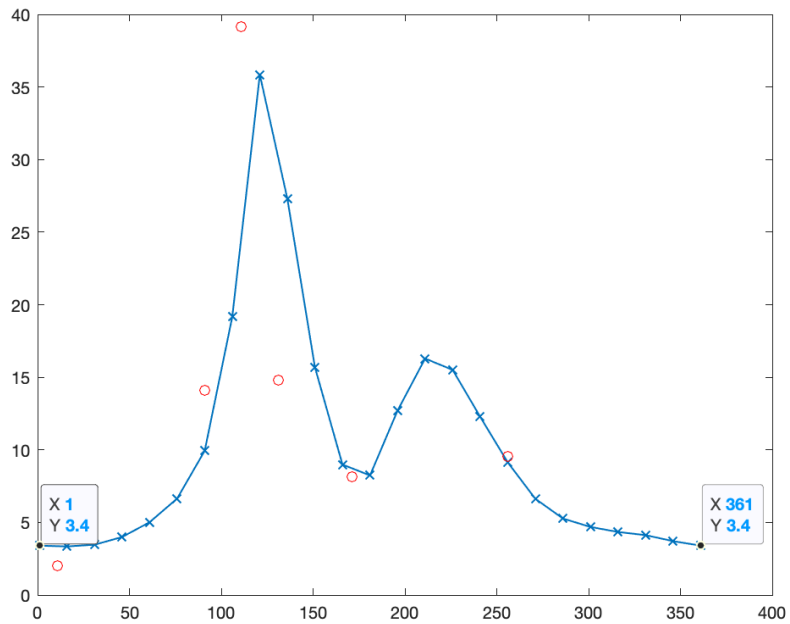
Figure 7. approximate grazing coefficient g= `0.007434768` to generate a periodic solution for P by equating endpoints of distribution.

Figure 8. 10-year plot of P for the standard values used in Riley, except that we now use the approximate grazing coefficient g= `0.007434768` to generate a periodic solution for P. This will be our base case with which we compare some of the P distributions in the presence of randomized predator forcing Z.
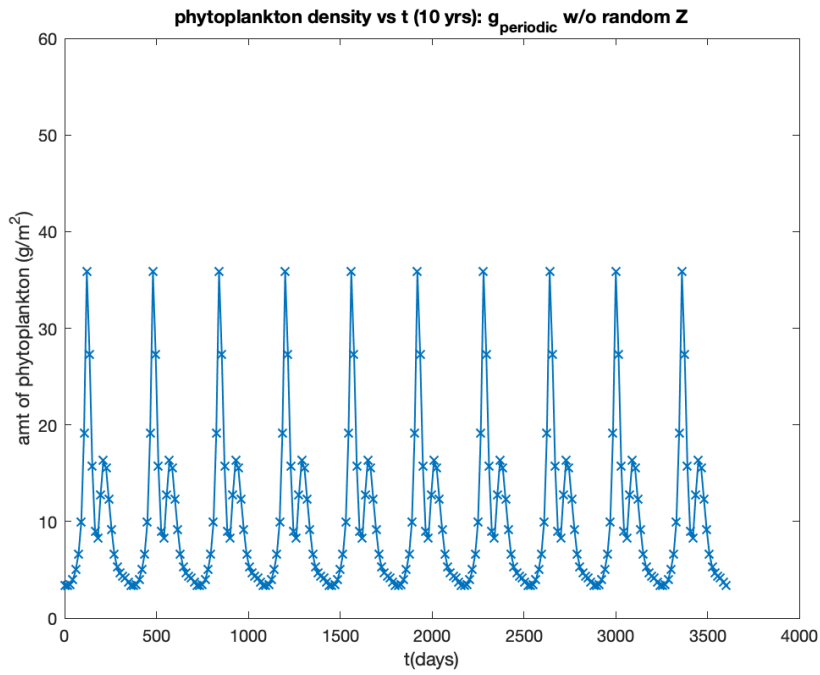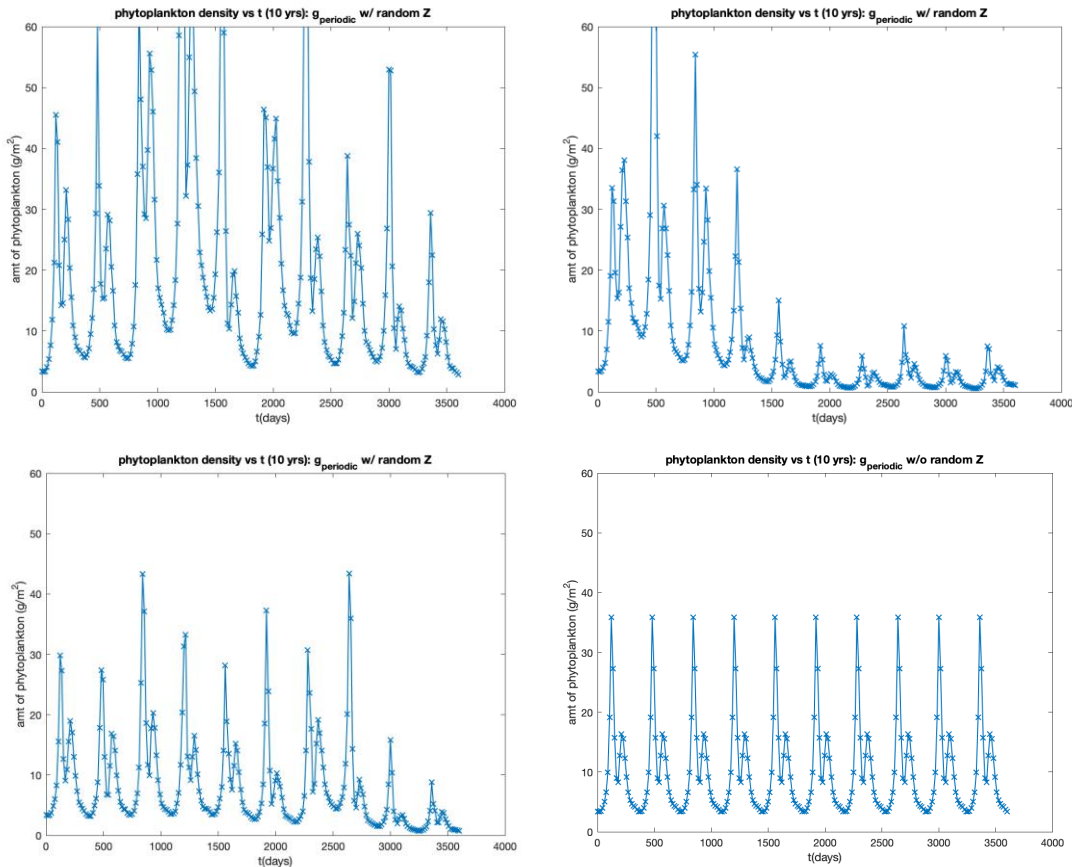
Figure 9. 10-year plot of P for the standard values used in Riley, except that we now use the approximate grazing coefficient g= `0.007434768` and randomized Z forcing. The bottom right diagram is the periodic non-randomized Z with the above grazing parameter to allow for periodic P solutions. The top left diagram shows the case where the random components on the whole act to diminish the zooplankton population enough that the P density is amplified and the signal becomes sharper. The bottom left demonstrates a similar phenomenon in the sense that there becomes an aperiodic intermittent signal averaging the original non-randomized P plot, but diverges from the top left in that P eventually is eaten down and the P population signal diminishes. The top right demonstrates an even steeper zooplankton forcing where it starts weak so P can increase significantly, then Z becomes stronger so that P decays midway through year 4.

Code for generating the above figures/sensitivity analysis: (green paragraphs indicate responses to questions)

```matlab
clear all; close all; fclose all;
cd('/Users/michaeldotzel/Desktop/12823');

load('Riley_dat.mat');
%In Riley (1946), we consider a simple plankton model P_t=P*(P_h-R-G),
%where P_h denotes the rate of photosynthesis (given as aan exponentially
%decaying function of depth, along with a vertical motion and nutrient
%depletion factors), R the effect of temperature, and G the grazing rate.
%Riley takes the grazing rate to be directly proportional to the predator
%density, and R exponential in temperature. Combining these somewhat
empirically
%motivated assumptions gives the final form
%
%dP/dt = P[(p*I_0/(k*z_1))*(1-exp(-k*z_1))*(1-N)*(1-V)-R_0*exp(r*T)-g*Z].
%
%Integrating this equation in time (for a series of relatively shorter, 15
day
%time intervals) yields the expression
%
%ln[P(t+15)/P(t)]=15*[P_h(t)-R(t)-G(t)],
%
%where here we assume that rates of growth on these small time intervals
%are (roughly) constant (hence our multiplication by t_f-t_i = 15).
%
%Out parameter values are
%
p=2.5;
R_0=0.0175;
r=0.069;
g=0.0075;
%
%In addition, we have initial condition that
%
P=NaN(25,1);
P(1)=3.4;

for i=1:length(T)
P(i+1)=P(i)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0*exp(r*T(i))-g*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
plot(t,P,'-x'); %plots the theoretical P with t

%here we shifted our initial points forward 1 day so that P_initial = P(1)
%instead of P(0) (so that indices jive with matlab).

%overlay with plot of observed P w/ resp to observed t (shift forward one
%day in the same vein as above theoretical estimate)
Obs_time=1+Obs_time; %since we are taking P(initial)=P(1) instead of P(0)
hold on; plot(Obs_time,Obs_P,'o','Color','r');
```

```
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('yearly phytoplankton density vs t (year theory and obs comparison)');

%We observe that the distribution obtained from this plot for the most part
%appears to duplicate that of Riley, with some small differences: the
%observational and theoretical primary peaks appear to not quite coincide
%as they seem to in his diagram, and while one might assume this is due to
%some kind of shifting error in the code, applying a shift of 10 days
%(which would position the observational peak point over the theoretical
%peak) is not enough to bring the remaining observations to coincide exactly
with
%the theoretical curve in the way depicted in Riley.
%%% Now we perform sensitivity analysis on each parameter
%
%For each parameter as listed above, we are interested in how dependent the
%phytoplankton are to changes to their growth dynamics. First we determine
%the sensitivity of P to the rate of photosynthesis amplifying parameter p:
p_vary=[2 2.25 2.5 2.75 3];
R_0=0.0175;
r=0.069;
g=0.0075;
P=NaN(25,5);
    P(1,:)=3.4;
for j=1:length(p_vary)
for i=1:length(T)
P(i+1,j)=P(i,j)*exp(15*[(p_vary(j)*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0*exp(r*T(i))-g*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P(:,j),'-x','LineWidth',2); %plots the theoretical P with t
 else
    plot(t,P(:,j),'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t: vary p');
legend('p=2.25',...
    ['p=2.5 (original value)'],['p=2'],['p=2.75'],['p=3']);
%
%
%We first observe that at 20% enhancement of p, the solution for P becomes
%very large, with values more than 5x larger at the first local maximum in
%P than the p=2.5 Riley case, and nearly 60x larger at the 2nd local
%maximum than the original. It is also worth noting that the second local
%maximum which was smaller than the first in the original p case, becomes
%more recessive with smaller p while increasing to overtake the value of P
%at the first local maximum for larger p. This could be attributed to the
%enhancement of light intensity in the northern hemisphere (e.g. the Georges
Bank region)
%during the summer and the corresponding emphasis on light exposure with
```

```matlab
%increasing p. For smaller p, temperature-dependent reduction in growth
%rate becomes more important for the higher temperatures in summer than in
%spring. As I am sure will be important for the second part of the
%assignment, it is as well interesting to see that for p even 10% larger,
%P does not oscillate seasonally - instead, there is an annual net
%increase.
%
%Now we determine the sensitivity of P to the temperature-dependent growth
amplifying parameter R_0:
p=2.5;
R_0vary=[0.014 0.01575 0.0175 0.01925 0.021];
r=0.069;
g=0.0075;
P=NaN(25,5);
    P(1,:)=3.4;

for j=1:length(R_0vary)

for i=1:length(T)
P(i+1,j)=P(i,j)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0vary(j)*exp(r*T(i))-g*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P(:,j),'-x','LineWidth',3); %plots the theoretical P with t
 else
   plot(t,P(:,j),'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t: vary R_0');
legend('R_0=0.014',...
    ['R_0=0.01575'],['R_0=0.0175 (original
value)'],['R=0.01925'],['R_0=0.021']);
%
%
%Our phytoplankton model is less sensitive to the effects of changing the
%amplitude of the temperature-dependent term than changing p
%(though indeed still sensitive). I say this for two reasons: the general
%qualitative features of a sharper local maximum followed by a smaller
%secondary local maximum are preserved, and the end-of-year deviation from
%the initial P density is much smaller in magnitude than that observed when
%varying p. The original value of R_0 (in yellow) in fact provides an
%close-to-periodic-oscillation of P, with P(end)=2.925 for
%P(initial)=3.4. When decreasing R_0 10% our P distribution no longer
%promises anything in the vicinity of periodicity, with P(final) being 3
%times larger than P(initial), and decreasing 20% P(final) becomes 10x
%larger than P(initial). However, this suggests that (because computed
%values of P are continuous with respect to a continuous parameter change)
%there is a value of R_0 which is slightly smaller than 0.0175 (original)
%which does provide oscillation of P for fixed other parameters. When R_0
```

```
%increases there is further dissipation of P, with P(final) becoming much
%smaller than P(initial). These features are all consistent with the
%temperature-dependent term interpretation of Riley's model: chiefly a
%decay term, for smaller R_0 there is less decay due to temperature, hence
%a relative increase in the amplitude of the second peak. For larger R_0
%however, in the warmer summer months temperature plays a much more
%enhanced role in depressing P growth, hence the much smaller secondary
%peak and in the end a net negative yearly effect on P.
%
%Now we determine the sensitivity of P to the exponential temperature rate of
increase r:
p=2.5;
R_0=0.0175;
r_vary=[0.0552 0.0621 0.069 0.0759 0.0828];
g=0.0075;
P=NaN(25,1);
P(1)=3.4;


for j=1:length(r_vary)
for i=1:length(T)
P(i+1)=P(i)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0*exp(r_vary(j)*T(i))-g*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P,'-x','LineWidth',3); %plots the theoretical P with t
 else
   plot(t,P,'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t: vary r');
legend('r=0.0552',...
    ['r=0.0621'],['r=0.069 (original value)'],['r=0.0759'],['r=0.0828']);
%
%
%Our phytoplankton model is even less sensitive to the effects of changing
the
%exponent of the temperature-dependent term than changing R
%(though indeed still sensitive). I say this again for two reasons: the
general
%qualitative features of a sharper local maximum followed by a smaller
%secondary local maximum are preserved (as in the case of varying R), and
%the end-of-year deviation from
%the initial P density is even smaller in magnitude than that observed when
%varying R or P. When decreasing r 10% our P distribution also still fails to
%be periodic, however P(final) is still closer to P(initial) than that found
%for the corresponding value of R_0. Decreasing 20% P(final) becomes ~5x
%larger than P(initial). However, this suggests that (because computed
%values of P are continuous with respect to a continuous parameter change)
%there is a value of r which is slightly smaller than 0.069 (original)
```

```matlab
%which does provide oscillation of P for fixed other parameters. When r
%increases there is further dissipation of P, with P(final) becoming much
%smaller than P(initial). These features are all again consistent with the
%temperature-dependent term interpretation of Riley's model: chiefly a
%decay term, for smaller r there is exponential decrease of this term due to
temperature, hence
%a relative increase in the amplitude of the second peak. For larger r
%in the warmer summer months temperature plays a much more
%enhanced role in depressing P growth, hence the much smaller secondary
%peak and in the end a net negative yearly effect on P.
%
%Now we determine the sensitivity of P to grazing coefficient g:
p=2.5;
R_0=0.0175;
r=0.069;
g_vary=[0.006 0.00675 0.0075 0.00825 0.009];
P=NaN(25,1);
    P(1)=3.4;
for j=1:length(g_vary)
for i=1:length(T)
P(i+1)=P(i)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0*exp(r*T(i))-g_vary(j)*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P,'-x','LineWidth',3); %plots the theoretical P with t
 else
   plot(t,P,'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t: vary g');
legend('g=0.006',...
    ['g=0.00675'],['g=0.0075 (original value)'],['g=0.00825'],['g=0.009']);
%
%Our phytoplankton model is quite sensitive to the effects of changing the
%grazing coefficient. The reason is because qualitatively, the secondary
% local maximum in P (just the opposite of the varying p case) becomes even
% larger than the first local maximum when decreasing g more than 10% of the
original grazing
%coefficient value, whereas originally the first peak is more pronounced
%than the second (this property still holds when g is increased). However,
%the value of P at the secondary peak when decreasing g by 20% is close to
%20x that of the original P at this peak. This suggests that grazing
%(along with light uptake term governed by p) is a principal factor in the
%model governing the growth of P, in contrast to temperature effects.
%For smaller g value, less grazing of P by Z allows P to multiply faster
%than the predators can regulate them, so a net increase in P over the year
%follows, in addition to a larger spike in the summer months due to
%increased sunlight/larger rate of P growth relative to grazing. For
%g 10% less than the one originally used by Riley, already the peaks seem
%about even with one another in the spring as the summer, though still
```

```matlab
%coming out with a net positive annual increase in P despite Z's best
efforts.
%Of course if g is too large then P just plummets from overeating, which is
%what can be observed from the smaller g values than the original.
%
%
%Now we determine the sensitivity of P to the nutrient depletion rate
%factor 1-N. It is redundant to consider cases where these values are
uniformly increased or uniformly decreased by 10%
%or 20%, since this effect would be equivalent to changing p by that
%amount which was already analyzed. Therefore, in this case P is equally
sensitive to 1-N as p. We can instead think about performing sensitivity
%analysis for one 15 day period (i.e. for one time interval in which 1-N is
%constant). It is unlikely that such an analysis will yield anything striking
about the sensitivity of the forcing function
%itself - since we are leaving the parameters for all other times
%invariant, the qualitative difference between distributions will not
%differ appreciably. We've confirmed this below, in which we have increased
%1-N at each time interval separately by 20% to see how sensitive P is to
%1-N for one interval parameter change:
%

p=2.5;
R_0=0.0175;
r=0.069;
g=0.0075;
for j=1:24
    if j==1
        OMN(j,:)=[1.2*OneMinusN(1),OneMinusN(2:24)'];
    else
OMN(j,:)=[OneMinusN(1:j-1)',1.2*OneMinusN(j),OneMinusN(j+1:end)'];
    end
end
P=NaN(25,1);
    P(1)=3.4;
for j=1:size(OMN,1)
for i=1:length(T)
P(i+1)=P(i)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OMN(j,i)*OneMinusV(i)-R_0*exp(r*T(i))-g*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
figure(1)
   plot(t,P,'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t: vary nutrient depletion term 1-N for one
time interval');
%
%
%Now we will fit the observed P to the predicted P for each of the
%sensitivity studies performed above. We use a standard deviation method
first to
```

```matlab
%find the error for each theoretical sensitivity test relative to the
observation.
%We will also use an averaged relative error formula to fit the observations
and corresponding theoretical
%predictions. The reason I do so is twofold: 1) I did not catch that
%average error referred to standard deviation in the paper until rather
%late, and so then tried to resolve the ambiguity by another way; 2) though
slightly
%different, this average error method captures a similar local percent error
between theory
%and observation (in a somewhat weighted sense) as captured by the standard
%deviation.
%
%
%p_vary case (repeating part of code above)
p_vary=[2 2.25 2.5 2.75 3];
R_0=0.0175;
r=0.069;
g=0.0075;
P=NaN(25,5);
    P(1,:)=3.4;
for j=1:length(p_vary)
for i=1:length(T)
P(i+1,j)=P(i,j)*exp(15*[(p_vary(j)*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0*exp(r*T(i))-g*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P(:,j),'-x','LineWidth',2); %plots the theoretical P with t
 else
   plot(t,P(:,j),'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t: vary p');
legend('p=2',...
    ['p=2.25'],['p=2.5 (original value)'],['p=2.75'],['p=3']);

%now find corresponding value at observed times
a=1:1:24;
m=NaN(length(Obs_time),length(p_vary));
Pre_P=NaN(length(Obs_time),length(p_vary));
for i=1:length(Obs_time)
    F=find(Obs_time(i)>=t(a) & Obs_time(i)<=t(a+1)); %finds 15 day interval
in which observation is located
    if length(F)>1
        Pre_P(i,:)=P(F(2),:);
    else
        for j=1:size(P,2)
    m(i,j)=(P(F+1,j)-P(F,j))./(t(F+1)-t(F));
    Pre_P(i,j)=P(F,j)+(Obs_time(i)-t(F)).*m(i,j);
```

```matlab
        end
    end
end
%METHOD 1: standard deviation
ObsP=repmat(Obs_P',1,5);
Err=ObsP-Pre_P; %relative errors
SD=NaN(5,1);
for i=1:5
SD(i)=std(Err(:,i)); %for each sensitivity test, over all time points
end
SS=SD'./mean(Pre_P,1); %yields percent deviation of P from the predicted
value (Incorrect - not paper value)

%I do not obtain the right percentage error in this computation compared to
%that of Riley. I would at some point like to clear this up with you
%sometime, if alright - I somehow misinterpret the standard deviation
%method. I use an alternate method below, but cannot be nicely compared to
%that of Riley.

%METHOD 2: find relative error (%error=(obs-exp)/exp)
ObsP=repmat(Obs_P',1,5);
err=ObsP-Pre_P;
N=NaN(5,1);
N(1,1)=norm(err(:,1))/norm(Obs_P);
N(2,1)=norm(err(:,2))/norm(Obs_P);
N(3,1)=norm(err(:,3))/norm(Obs_P);
N(4,1)=norm(err(:,4))/norm(Obs_P);
N(5,1)=norm(err(:,5))/norm(Obs_P);
%N =

%     0.8500
%     0.6829
%     0.4613
%     2.2820
%    21.0114

%Note that of these relative errors for each varied value of p in our
%sensitivity analysis, the smallest in magnitude is that which corresponds
%to the initial choice of parameters, and which gives 46.13% averaged
relative error
%between the theoretical prediction and the observations.

%2. R_0 case
p=2.5;
R_0vary=[0.014 0.01575 0.0175 0.01925 0.021];
r=0.069;
g=0.0075;
P=NaN(25,5);
    P(1,:)=3.4;
for j=1:length(R_0vary)

for i=1:length(T)
P(i+1,j)=P(i,j)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0vary(j)*exp(r*T(i))-g*Z(i)]);
end
```

```matlab
%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P(:,j),'-x','LineWidth',3); %plots the theoretical P with t
 else
    plot(t,P(:,j),'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t: vary R_0');
legend('R_0=0.014',...
    ['R_0=0.01575'],['R_0=0.0175 (original
value)'],['R=0.01925'],['R_0=0.021']);

%now find corresponding value at observed times
a=1:1:24;
m=NaN(length(Obs_time),length(R_0vary));
Pre_P=NaN(length(Obs_time),length(R_0vary));
for i=1:length(Obs_time)
    F=find(Obs_time(i)>=t(a) & Obs_time(i)<=t(a+1)); %finds 15 day interval
in which observation is located
    if length(F)>1
        Pre_P(i,:)=P(F(2),:);
    else
        for j=1:size(P,2)
    m(i,j)=(P(F+1,j)-P(F,j))./(t(F+1)-t(F));
    Pre_P(i,j)=P(F,j)+(Obs_time(i)-t(F)).*m(i,j);
        end
    end
end

%METHOD 2: take relative error (%error=(obs-exp)/exp)
ObsP=repmat(Obs_P',1,5);
err=ObsP-Pre_P;
N=NaN(5,1);
N(1,1)=norm(err(:,1))/norm(Obs_P);
N(2,1)=norm(err(:,2))/norm(Obs_P);
N(3,1)=norm(err(:,3))/norm(Obs_P);
N(4,1)=norm(err(:,4))/norm(Obs_P);
N(5,1)=norm(err(:,5))/norm(Obs_P);

%N =

%    1.0526
%    0.5837
%    0.4613
%    0.5075
%    0.5927

%Again note that of these relative errors for each varied value of p in our
%sensitivity analysis, the smallest in magnitude is that which corresponds
%to the initial choice of parameters, and which gives 46.13% relative error
```

```matlab
%between the theoretical prediction and the observations. An increase to 110%
of the
%original R_0 value produces a somewhat closer relative error estimate of
50.75%.


%3. Vary exponential temperature rate of increase r
p=2.5;
R_0=0.0175;
r_vary=[0.0552 0.0621 0.069 0.0759 0.0828];
g=0.0075;
P=NaN(25,5);
P(1,:)=3.4;

for j=1:length(r_vary)
for i=1:length(T)
P(i+1,j)=P(i,j)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0*exp(r_vary(j)*T(i))-g*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P(:,j),'-x','LineWidth',3); %plots the theoretical P with t
 else
    plot(t,P(:,j),'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t: vary r');
legend('r=0.0552',...
    ['r=0.0621'],['r=0.069 (original value)'],['r=0.0759'],['r=0.0828']);

%
a=1:1:24;
m=NaN(length(Obs_time),length(R_0vary));
Pre_P=NaN(length(Obs_time),length(R_0vary));
for i=1:length(Obs_time)
    F=find(Obs_time(i)>=t(a) & Obs_time(i)<=t(a+1)); %finds 15 day interval
in which observation is located
    if length(F)>1
        Pre_P(i,:)=P(F(2),:);
    else
        for j=1:size(P,2)
    m(i,j)=(P(F+1,j)-P(F,j))./(t(F+1)-t(F));
    Pre_P(i,j)=P(F,j)+(Obs_time(i)-t(F)).*m(i,j);
        end
    end
end

%take relative error (%error=norm(obs-exp)/norm(exp))
ObsP=repmat(Obs_P',1,5);
err=ObsP-Pre_P;
```

```matlab
N=NaN(5,1);
N(1,1)=norm(err(:,1))/norm(Obs_P);
N(2,1)=norm(err(:,2))/norm(Obs_P);
N(3,1)=norm(err(:,3))/norm(Obs_P);
N(4,1)=norm(err(:,4))/norm(Obs_P);
N(5,1)=norm(err(:,5))/norm(Obs_P);
%N =

%    0.5469
%    0.4759
%    0.4613
%    0.4711
%    0.4883


%Here we observe that just as in the other sensitivity analyses, of each case
the
%original conditions produce the smallest average relative error at 46.13%.
%
%
%4. vary grazing constant g
p=2.5;
R_0=0.0175;
r=0.069;
g_vary=[0.006 0.00675 0.0075 0.00825 0.009];
P=NaN(25,5);
    P(1,:)=3.4;
for j=1:length(g_vary)
for i=1:length(T)
P(i+1,j)=P(i,j)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0*exp(r*T(i))-g_vary(j)*Z(i)]);
end


%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P(:,j),'-x','LineWidth',3); %plots the theoretical P with t
 else
    plot(t,P(:,j),'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t: vary g');
legend('g=0.006',...
    ['g=0.00675'],['g=0.0075 (original value)'],['g=0.00825'],['g=0.009']);
%
a=1:1:24;
m=NaN(length(Obs_time),length(g_vary));
Pre_P=NaN(length(Obs_time),length(g_vary));
for i=1:length(Obs_time)
    F=find(Obs_time(i)>=t(a) & Obs_time(i)<=t(a+1)); %finds 15 day interval
in which observation is located
    if length(F)>1
        Pre_P(i,:)=P(F(2),:);
```

```
    else
        for j=1:size(P,2)
    m(i,j)=(P(F+1,j)-P(F,j))./(t(F+1)-t(F));
    Pre_P(i,j)=P(F,j)+(Obs_time(i)-t(F)).*m(i,j);
        end
    end
end

%take relative error (%error=(obs-exp)/exp)
ObsP=repmat(Obs_P',1,5);
err=ObsP-Pre_P;
N=NaN(5,1);
N(1,1)=norm(err(:,1))/norm(Obs_P);
N(2,1)=norm(err(:,2))/norm(Obs_P);
N(3,1)=norm(err(:,3))/norm(Obs_P);
N(4,1)=norm(err(:,4))/norm(Obs_P);
N(5,1)=norm(err(:,5))/norm(Obs_P);

%N =

%    3.7451
%    0.9746
%    0.4613
%    0.5362
%    0.6558

%Here we observe that just as in the other sensitivity analyses, of each case
the
%original conditions produce the smallest relative error at 46.13%.
%Increasing g to 110% of the original value yields P which is relatively
close to
%that of the original parameter values, with relative error 53.62%.
%
%In summary, the relative error method gives a relative measure of how
%important parameters are to the P solution by considering how different
%the errors are between the original parameter value-determining P and the
%plus/minus 10% or 20% the value of the respective parameter on P.
%In this way, one can observe that P is extremely sensitive to p,
%moderately sensitive to both g and R_0, and not as sensitive to r.
%


%We now turn our attention to the case where P is periodic, i.e. if there
%is annual cycling of P. In order for this to happen, we must have
%P(end)=P(initial)=3.4. We observed that in all our sensitivity and
%original analyses, we have come close to periodic oscillation in P but not
%quite. We will then change the grazing parameter and keep the other
%parameters constant to see what value of g yields this assumption
%(assuming of course that other parameter values are also the same constant
%values for their corresponding time interval of the year):

p=2.5;
R_0=0.0175;
r=0.069;
g_vary=[0.0074 0.00745 0.0075 0.00755 0.0076];
```

```
P=NaN(25,5);
    P(1,:)=3.4;
for j=1:length(g_vary)
for i=1:length(T)
P(i+1,j)=P(i,j)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0*exp(r*T(i))-g_vary(j)*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P(:,j),'-x','LineWidth',3); %plots the theoretical P with t
 else
   plot(t,P(:,j),'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t: vary g');
legend('g=0.0074',...
    ['g=0.00745'],['g=0.0075 (original)'],['g=0.00755'],['g=0.0076']);

%As we can observe in the corresponding figure, there is periodic P
%behavior for the grazing parameter approximate value g=0.0074348, which is
less than 1% from
%the original g=0.0075 value used by Riley. Now of course, if one were to
%increase p or decrease R_0 or r, one would expect the necessary value of g
%would increase to compensate. This is demonstrated below, for increasing p
%to 110% of its original value:

p=2.75;
R_0=0.0175;
r=0.069;
g_vary=[0.008 0.00825 0.0085 0.00875 0.009];
P=NaN(25,5);
    P(1,:)=3.4;
for j=1:length(g_vary)
for i=1:length(T)
P(i+1,j)=P(i,j)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0*exp(r*T(i))-g_vary(j)*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P(:,j),'-x','LineWidth',3); %plots the theoretical P with t
 else
   plot(t,P(:,j),'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
```

```matlab
title('phytoplankton density vs t: vary g');
legend('g=0.008',...
    ['g=0.00825'],['g=0.0085'],['g=0.00875'],['g=0.009']);

%As can be observe in this figure, the g value that would yield a periodic
%solution of P is closer to g=0.0087. This is consistent with what we would
expect:
%as we are changing g in response to other parameters, p increasing would
%have the same effect as decreasing either r or R_0 and would boost the P
%density, which must then be depressed by an increase in g for yearly
%oscillation of P to occur.

%
p=2.5;
R_0=0.0175;
r=0.069;
g_vary=0.007434768;
P=NaN(25,5);
    P(1,:)=3.4;
for j=1:length(g_vary)
for i=1:length(T)
P(i+1,j)=P(i,j)*exp(15*[(p*I(i)/(k(i)*z1(i)))*(1-exp(-
k(i)*z1(i)))*OneMinusN(i)*OneMinusV(i)-R_0*exp(r*T(i))-g_vary(j)*Z(i)]);
end

%P=P(1:15:end);
t=1:15:361;
figure(1)
 if j==3
 plot(t,P(:,j),'-x','LineWidth',3); %plots the theoretical P with t
 else
    plot(t,P(:,j),'-x','LineWidth',1); %plots the theoretical P with t
hold on
end
end
hold on; plot(Obs_time,Obs_P,'o','Color','r');
%
%
%Now we vary the predator population by introducing a random component to
%the Z observations. For a 10 year timeseries, we assume that Z without the
%random component is
Zrand=NaN(10*length(Z),1);
for i=1:length(Zrand)
    Zrand(i)=rand;
end
ZZ=repmat(Z,10,1);
Zr=ZZ.*(0.8+0.4*Zrand); %defines new Z population

TT=repmat(T,10,1);
OMV1=repmat(OneMinusV,10,1);
OMN1=repmat(OneMinusN,10,1);
II=repmat(I,10,1);
Z1=repmat(z1,10,1);
KK=repmat(k,10,1);

p=2.5;
```

```matlab
t1=1:15:1+15*length(ZZ);
R_0=0.0175;
r=0.069;
g_vary=0.007434768;
P=NaN(1+length(ZZ),1);
    P(1)=3.4;
for i=1:length(TT)
P(i+1)=P(i)*exp(15*[(p*II(i)/(KK(i)*Z1(i)))*(1-exp(-
KK(i)*Z1(i)))*OMN1(i)*OMV1(i)-R_0*exp(r*TT(i))-g_vary*ZZ(i)]);
end
%P=P(1:15:end);
t=1:15:361;
figure(1)
    plot(t1,P,'-x','LineWidth',1); %plots the theoretical P with t
ylim([0 60]);
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t (10 yrs): g_{periodic} w/o random Z');
%plotting non-random Z


%Now for randomized Z:

Zrand=NaN(10*length(Z),1);
for i=1:length(Zrand)
    Zrand(i)=rand;
end
ZZ=repmat(Z,10,1);
Zr=ZZ.*(0.8+0.4*Zrand); %define random Z component
for i=1:length(TT)
P(i+1)=P(i)*exp(15*[(p*II(i)/(KK(i)*Z1(i)))*(1-exp(-
KK(i)*Z1(i)))*OMN1(i)*OMV1(i)-R_0*exp(r*TT(i))-g_vary*Zr(i)]);
end

%t=1:15:361;
figure(1)
    plot(t1,P,'-x','LineWidth',1); %plots the theoretical P with t
ylim([0 60]);
xlabel('t(days)');ylabel('amt of phytoplankton (g/m^2)');
title('phytoplankton density vs t (10 yrs): g_{periodic} w/ random Z');
%plotting non-random Z
hold on;

%It is easy to see that in the attached figures, the random aspect of the
zooplankton
%term can induce a lot of variability in P - in one case, the periodic
%signature simply seems to diminish in magnitude and dissipate over the 10
%years, while another displays quasi-periodic, intermittent pulsing of
%phytoplankton before dissipating, while yet the third plot shows an
%on-average weaker grazing component which manifests as a consistently
%larger and steeper P density plot.
```