Isabel Honda
SID# 911284851
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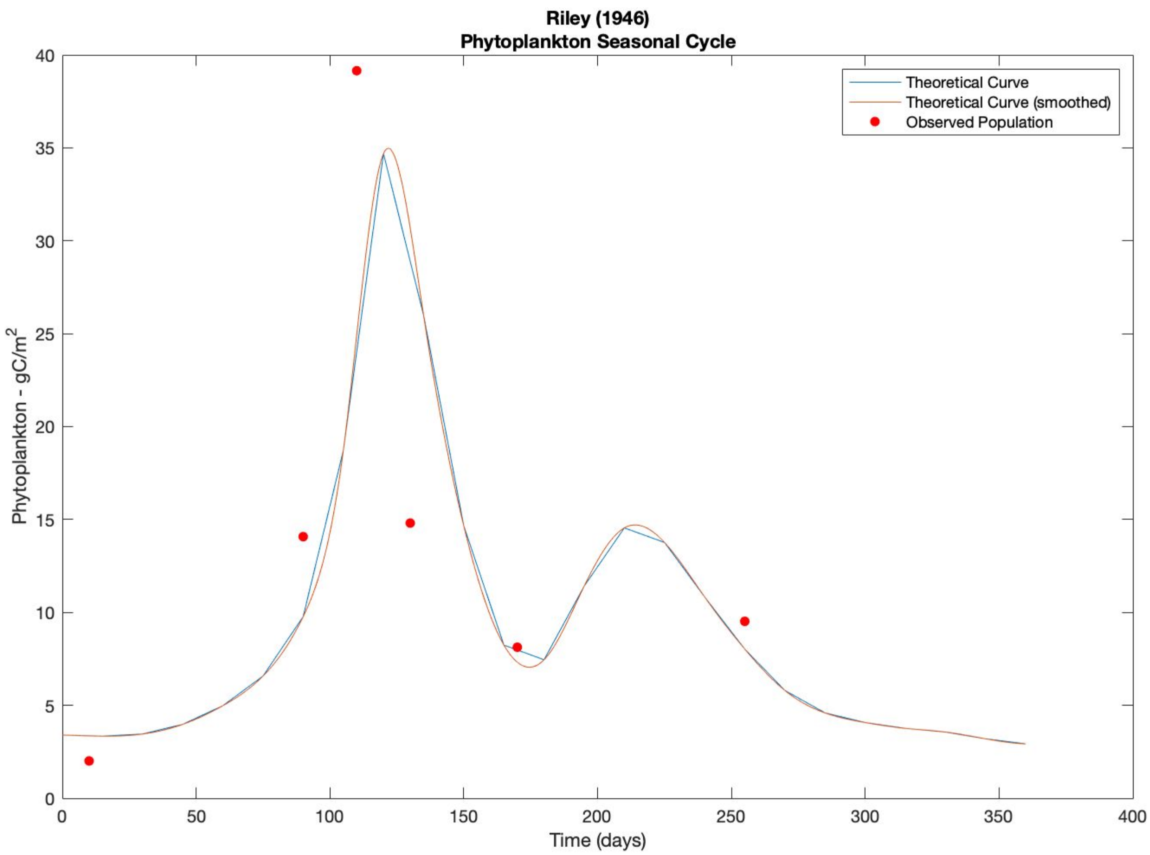
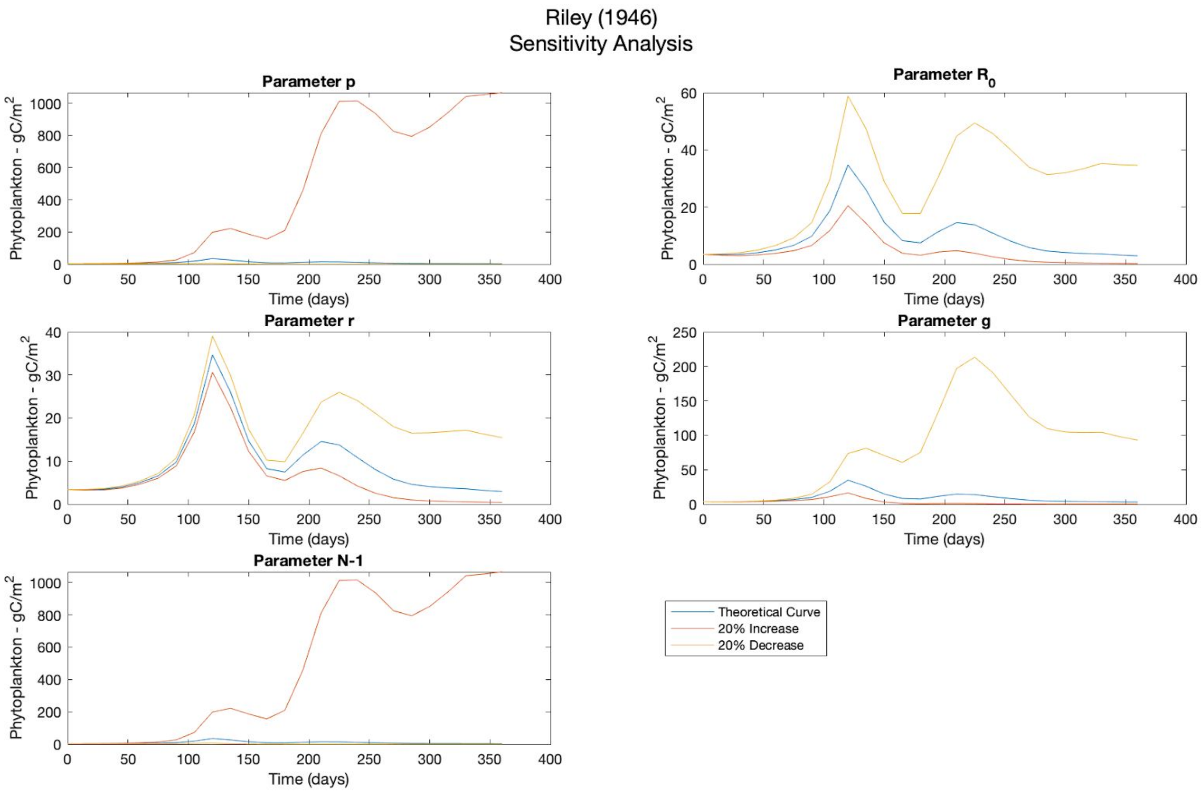
12.823 Problem Set 1

Problem 1: Implement Riley's 1946 Simple Plankton Model

- Run the model to verify you can reproduce Riley's results.
    - The results from running the model are displayed below. Both the unsmoothed direct output of the model and the smoothed output are plotted along with the observed population.



❖ Perform a sensitivity analysis and plot the results together with the baseline solution. Describe how each parameter variation affects the solution. What do you conclude about the relative sensitivity of the model solution to each parameter?



➢ As can be seen from these results, changing the parameters of the model can give dramatically different results in phytoplankton concentration, but the overall periodic pattern with two peaks can still generally be seen.

➢ A 20% increase in p and N-1, which are the parameters that affect the photosynthetic rate function ($P_h(t)$) results in an exponential increase in the theoretical output. This indicates that slight changes in the photosynthetic constant and phytoplankton nutrient depletion greatly impact the overall phytoplankton concentration.

➢ Changes to parameters R_0 and r affecting the phytoplankton respiration function ($R(t)$) give relatively modest changes in the theoretical results, with a 20% increase causing a decrease in the overall phytoplankton concentration. This makes intuitive sense considering that respiration reduces overall phytoplankton biomass.

➢ Finally, changes to parameter g, which affects the zooplankton grazing function ($G(t)$) can also have a massive effect on the final theoretical results, with a 20% decrease in this parameter resulting in a massive increase in phytoplankton concentration, whereas increasing this parameter by 20% results in a relatively modest decrease in phytoplankton concentration. Perhaps this could be explained by the fact that g is the rate of reduction of phytoplankton, so decreasing this parameter allows phytoplankton to grow much faster, since phytoplankton growth is governed by an exponential function.

- Evaluate the fit to the data in each of your sensitivity experiments, and compare to the 27% error that Riley computed for his simulation. Would you add any caveats to Riley's conclusion based on what you have found?
    - Comparing the fit to the data for the baseline model resulted in an error of 37.13%. The errors for each of the sensitivity experiments are given in the table below.

Parameter	**p**	**R_0**	**r**	**g**	**N-1**
20% Increase	0.73906	1.5987	0.89394	5.7569	0.73906
20% Decrease	28.768	0.41493	0.39706	0.53498	28.768

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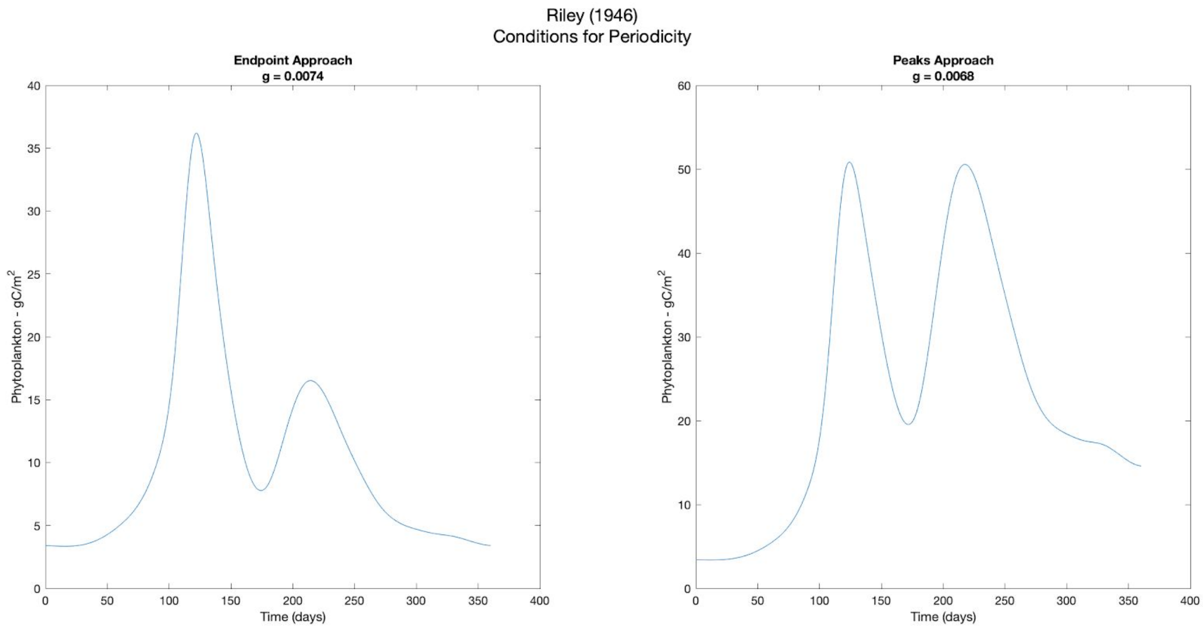
    - As can be seen from the errors above, changes to these parameters results in dramatically different errors compared to the observational data points.
    - Based on these findings, I would add the caveat that the 27% error is only accurate for the particular parameters that were given for the model, since a 20% variation drastically changes the error. Riley does concede in the Summary section of the paper that modeled values can deviate from the observations by about 20-40%. Since ocean systems and ecosystem dynamics are constantly fluctuating, it is likely that the processes that these given parameters represent tend to deviate from the particular values given in the model.
    - Increasing the number of observations would most likely result in more accurate model results and model parameters that are better fitted to real-world observations.

Problem 2: Periodic Conditions

- What are the conditions for P to be periodic? Find the g value that ensures this.
    - A function, $f(x)$, is periodic if, for all values of x, there exists a positive number $T \in \mathbb{R}$ such that $f(x+T) = f(x)$, where T is the period of $f(x)$ (UBC Math).
    - In this Riley (1946) example, there are several conditions that indicate that P is periodic.
        - One method (the "endpoint approach") is to find a value of g such that P at timestep 0 is equal to P at timestep 25. This indicates that the period is approximately one year and the cycle would repeat after one year.
        - Another method (the "peaks approach") is to find a value of g such that the two peaks in the data are equal in magnitude. In this case, these two peaks represent two complete cycles in a year.

- The graphs of P with these two approaches and their corresponding g values are shown below.



➢ As can be seen from these plots, both the endpoint approach and the peaks approach appear to result in P having a periodic pattern and output g values that are within 0.0001 and 0.0007 (respectively) of the provided g value.
   - The endpoint approach gives a g value of approximately 0.0074. This time series would need to be extended to multiple years in order to see more than one full period.
   - The peaks approach gives a g value of 0.0068. Since the peaks are asymmetric and the tail of the second peak appears to taper off at around 15 gC/m^2 instead of decreasing rapidly to P(0), this g value may not be the most optimal for ensuring periodicity.
      - Therefore, the following analysis will utilize the endpoint approach and the **g value of 0.0074**.

❖ How does this value vary as you change the other parameters?

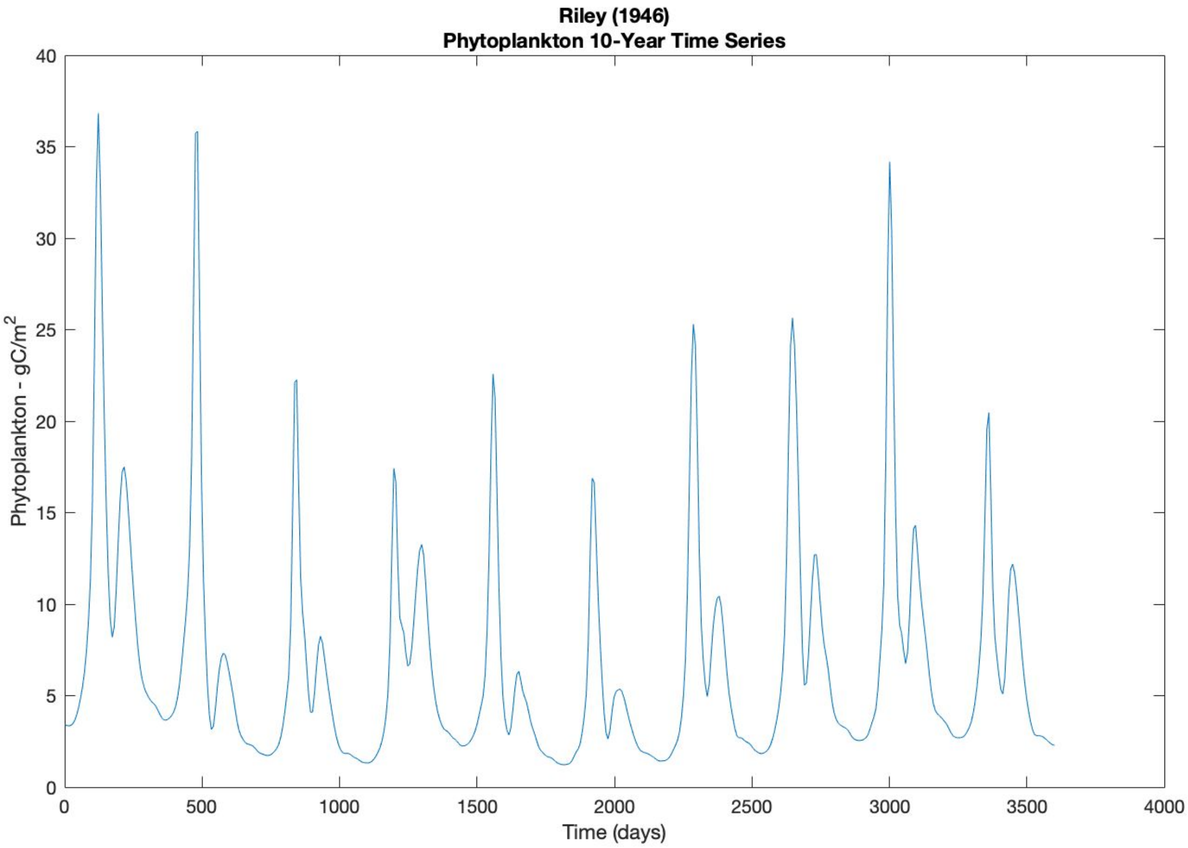
➢ The value of g varies depending on the values of the other parameters. Increasing and decreasing the following parameters by 20% gives the following corresponding g values (as determined via the endpoint approach).

Parameter	**p**	**R_0**	**r**
20% Increase	g = 0.01	g = 0.0064	g = 0.0066
20% Decrease	g = 0.0049	g = 0.0085	g = 0.0082

- ➢ These g values appear to oscillate around the provided g value of 0.0075, with a range of $\pm 2.6 \times 10^{-3}$. Changing the p parameter by $\pm 20\%$ alters the value of g the most dramatically.

Problem 3: 10-Year Series

- ❖ Construct a 10-year series allowing the Z values to vary randomly by 20%. Discuss.



- ➢ As can be seen from the plot above, letting the quantity of zooplankton vary randomly by 20% produces the modeled interannual variability, yet phytoplankton concentration still follows a periodic pattern with biannual peaks. For each year, the first and largest peak occurs in around late spring/early summer while the smaller second peak occurs in around late summer/early fall.
- ➢ At any value other than g $\approx$ 0.0074, the model does not oscillate periodically and either rapidly approaches zero or diverges exponentially.
- ➢ Due to top-down dynamics and the Moran effect, zooplankton quantities likely closely follow these phytoplankton populations and similarly have biannual peaks in abundance.

Works Cited

University of British Columbia Mathematics Department. (n.d.). *Periodic functions and boundary conditions*. Math.Ubc.Ca. https://www.math.ubc.ca/~njb/m256n6.pdf