

12.823 Problem Set #1
Assigned March 1, 2021
Due March 15
Canvas Site indicates March 29

(1) Implement Riley's (1946) simple plankton model

$$\frac{\partial P}{\partial t} = P(P_h - R - G)$$

using his form

$$\ln \frac{P(t + 15)}{P(t)} = 15 * [P_h(t) - R(t) - G(t)]$$

where time is in units of days, the time step is 15 days.

Run the model to verify you can reproduce Riley's results (Fig. 21). For your convenience, a file (Riley_dat.mat) containing the observations and his forcing functions (from the Appendix) is provided. In your model runs, make the same assumptions as Riley:

Initial condition: $P(t = 0) = 3.4 \text{ g C m}^{-2}$;

Parameter values:

$$p = 2.5 \text{ day}^{-1} (\text{g cal cm}^{-2} \text{ min}^{-1})^{-1}$$

$$R_0 = 0.0175 \text{ day}^{-1}$$

$$r = 0.069 (\text{deg C})^{-1}$$

$$g = 0.0075 \text{ day}^{-1} (\text{g C m}^{-2})^{-1}.$$

A year can be simplified as 12 30-day months.

Perform a sensitivity analysis to each of the following parameters: p , R_0 , r , g , and $1 - N$. For each parameter, do at least two trials in which the value of the parameter is increased by 20% and then decreased by 20%. Plot the results together with the baseline solution and describe how each parameter variation affects the solution. What do you conclude about the relative sensitivity of the model solution to each parameter?

Riley concludes that his dynamic model of the phytoplankton population yields results that are the same order of accuracy as the statistical estimate. Evaluate the fit to the data in each of your sensitivity experiments, and compare to the 27% error that Riley computed for his simulation. Would you add any caveats to Riley's conclusion based on what you have found?

(2) What are the conditions for P to be periodic? Find the g value that ensures this. How does this value vary as you change the other parameters?

(3) Start with the base case with g adjusted for periodicity. Construct a 10-year series allowing the Z values to vary randomly by 20%; i.e. $Z(t) = Z_{\text{Riley}}(t) * (0.8 + 0.4 \text{ rand})$. Discuss.

Please attach your matlab/octave code when you hand in your homework.